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NOTES ON THE RELATIONSHIP OF SCINTILLATION INDEX TO PROBABILITY DISTRIBUTIONS AND THEIR USES FOR SYSTEM DESIGN

Herbert E. Whitney

Air Force Cambridge Research Laboratories
L. G. Hanscom Field, Massachusetts

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13. ABSTRACT

The characterization of ionospheric scintillations in terms of scintillation index is discussed. Scintillation index cannot be directly applied to the design of systems that use transionospheric propagation. The relationships of scintillation index to system design parameters, cumulative amplitude distributions, and fade levels, are shown. The determination of a spectral index is explained and its use for extrapolating the results of ionospheric scintillation measurements to a desired frequency is shown.

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1. INTRODUCTION

Ionospheric scintillations can be a serious problem to many systems that transmit through the ionosphere and are sensitive to fading of the signal. Some communication systems are designed with as little as a 4-dB margin and scintillations can often cause this level to be exceeded even at mid-latitudes for VHF and UHF transmissions. Equatorial and auroral stations have shown greater changes at even higher frequencies. Much data that has been taken for characterizing scintillations is not in a form which can be directly used by the engineer for system design or the physicist interested in mechanisms governing the generation of scintillations. It is the intent of this note to show the relationships between scintillation indices and amplitude distributions and ways in which existing data can be made useful for systems' application. Since a given application will often be for a frequency which is different than that described by the data, a method is presented for scaling the data to other frequencies.

(Received for publication 2 January 1974)

2. SCINTILLATION INDEX

The morphology of ionospheric scintillations has been usefully described by scintillation indices that are a measure of the fluctuations imposed on a signal as it traverses the ionosphere. A simple index was adopted by AFCRL for uniformly scaling chart recordings of the strength of satellite beacons.[1,2] Scintillation index (SI) was defined as $(P_{max} - P_{min})/(P_{max} + P_{min})$, where P_{max} is the power amplitude of the third peak down from the maximum excursion of the scintillations and P_{min} is the power amplitude of the third level up from the minimum excursion in the sample period, typically 15 minutes. It was found that scaling the chart records was considerably simplified by just measuring the decibel change between P_{max} and P_{min}. By assuming equal percentage changes from the average level for P_{max} and P_{min}, a relationship between dB and percentage canbe determined. This is shown in Table 1 and Figure 1.

Table 1. Conversion Table: $P_{max} - P_{min}$(dB) vs SI (%)

-dB	+ dB	$P_{max} - P_{min}$(dB)	SI (%)
.09	.09	.18	2
.22	.21	.43	5
.46	.41	.87	10
.71	.61	1.32	15
.97	.79	1.76	20
1.25	.97	2.22	25
1.55	1.14	2.69	30
1.87	1.30	3.17	35
2.22	1.46	3.68	40
2.60	1.61	4.21	45
3.01	1.76	4.77	50
3.47	1.90	5.37	55
3.98	2.01	5.99	60
4.56	2.18	6.74	65
5.23	2.31	7.54	70
6.02	2.43	8.45	75
6.99	2.55	9.54	80
8.24	2.67	10.91	85
10.00	2.79	12.79	90
13.00	2.90	15.90	95
20.00	2.99	22.99	99
30.00	3.00	33.00	99.9

1. Whitney, H.E. and Malik, C. (1968) A Proposed Index for Measuring Ionospheric Scintillation, AFCRL-68-0138.
2. Whitney, H.E., Aarons, J. and Malik, C. (1969) A proposed index for measuring ionospheric scintillations, Planet. Space Sci., 17:1069-1073.

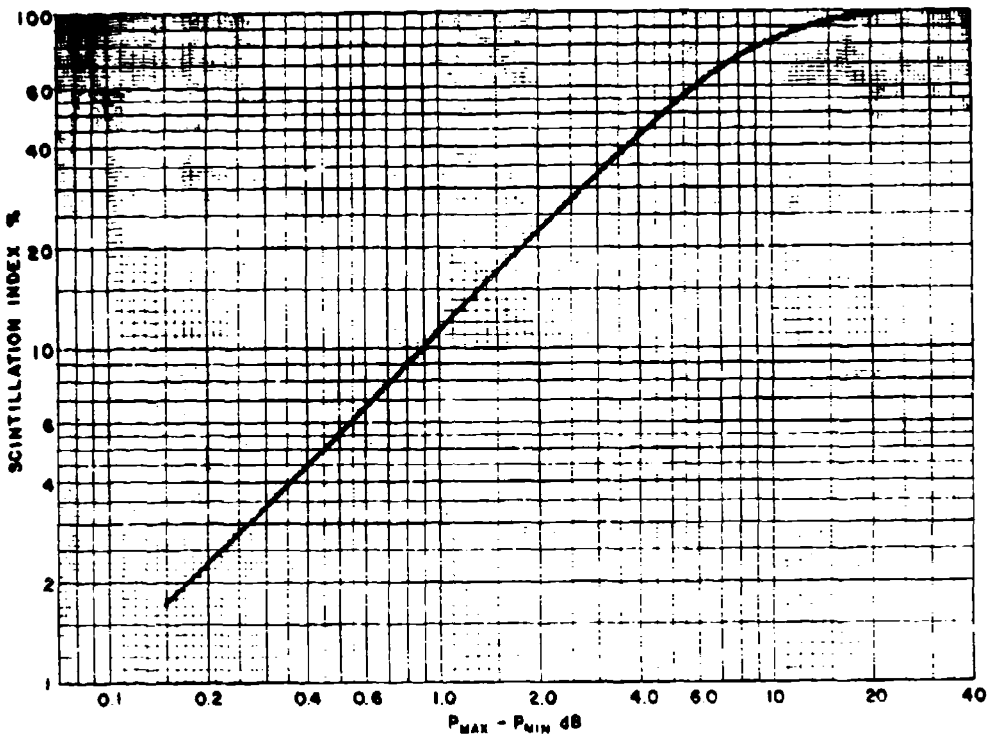

Figure 1. Graph for the Conversion of P_{max} - P_{min} to Scintillation Index. P_{max} - P_{min} is the peak to peak excursion of a scintillating signal and is measured in decibels based on an amplitude calibration of the chart record

3. CUMULATIVE AMPLITUDE PROBABILITY DISTRIBUTION FUNCTION

While the characterization of ionospheric scintillations has been extremely useful for describing the morphology of the irregularity structure, it has not described the fading characteristics in a form suitable for engineering design, such as the fading margin necessary to overcome scintillations as a function of local time, latitude, season, magnetic activity, and so forth. The cumulative amplitude probability distribution function (cdf) is a form of data that is readily applied to system design. The relationship between SI and cdf has been determined for a large number of experimental distributions of synchronous satellite signals at 136 MHz.[3] General agreement between the experimental distributions and the Nakagami m distribution was found; examples are shown in Figure 2. Figure 2

3. Whitney, H. E., Aarons, J., Allen, R. S. and Seemann, D. R. (1972) Estimation of the cumulative amplitude probability distribution function of ionospheric scintillations, Radio Science, 7:1095-1104.

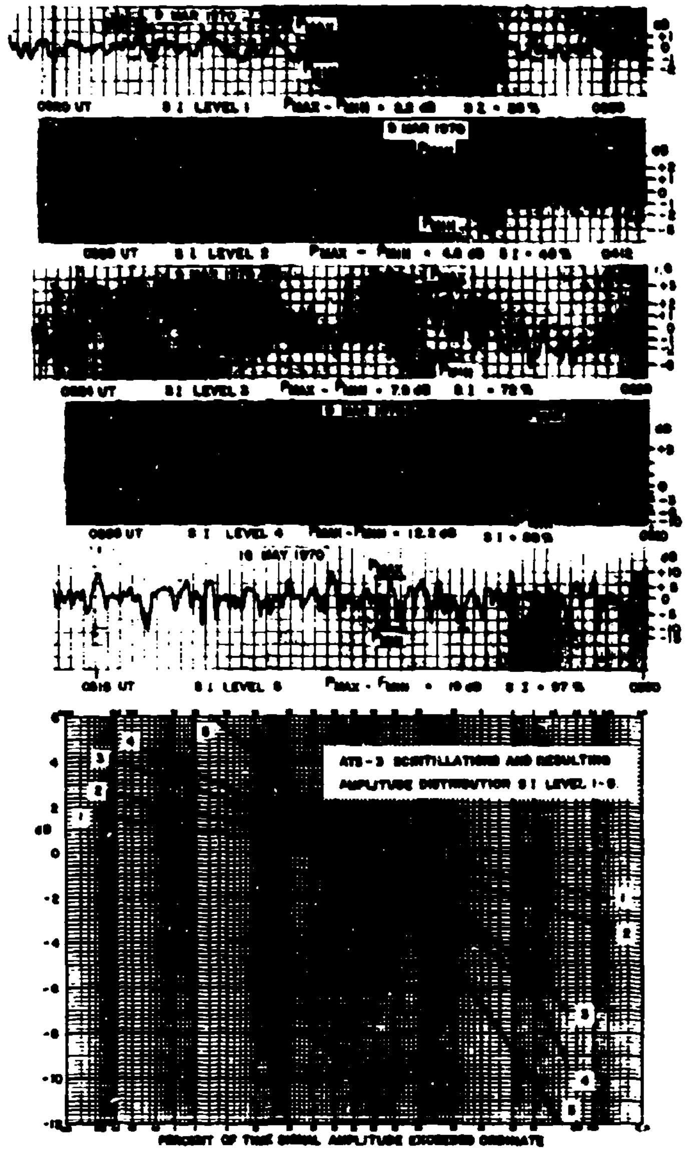

Figure 2. Examples of Ionospheric Scintillations at 136 MHz

also illustr_tes the scaling of scintillations for $P_{max} - P_{min}$ values. The punched card format for computer processing is given in Appendix A. Figure 3 shows the theoretical curves for the Nakagami distribution over a range of m values. If experime ta' data does not cover sufficient amplitude range, Figure 3 can be used to estimate xtreme values. Figure 4 shows the grouping of scintillation index values

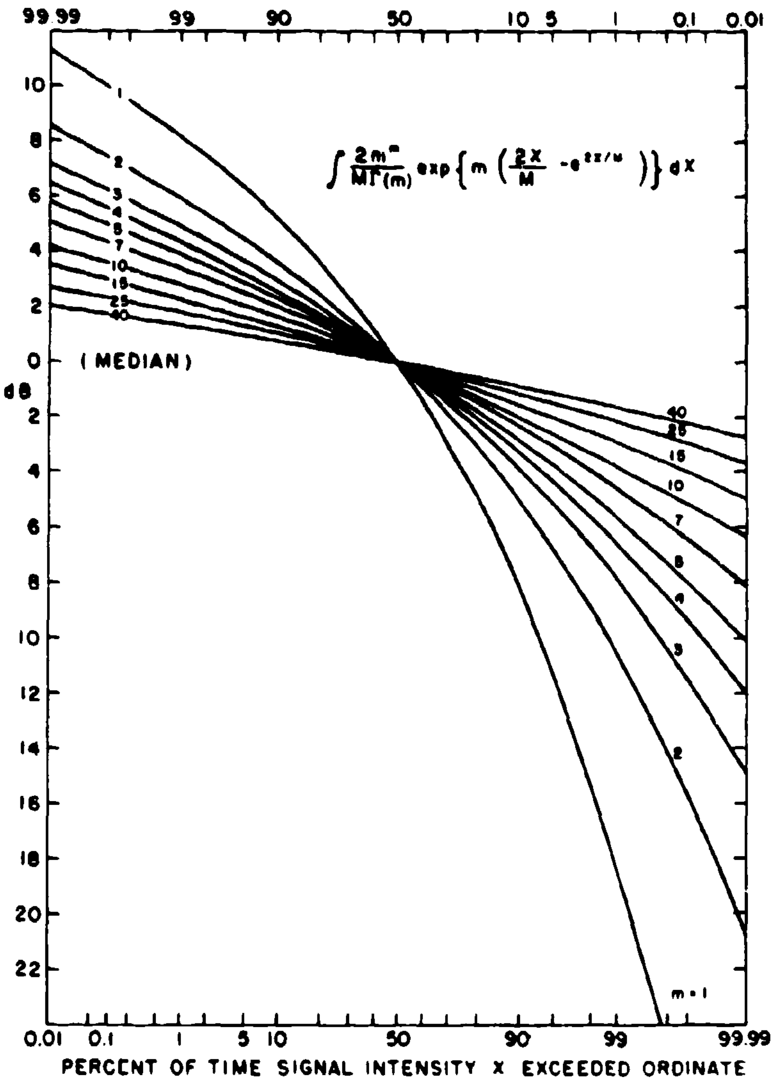

Figure 3. Theoretical Nakagami m Distribution

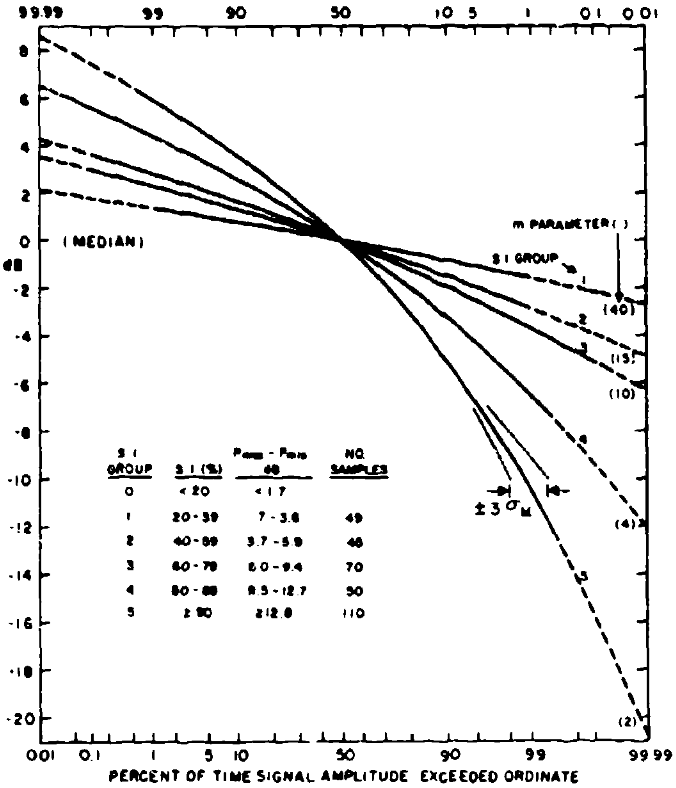

Figure 4. Model Distributions for Ionospheric Scintillations at 136 MHz

that was used to facilitate statistical studies and to generate the model distributions. The curves are median distributions for each SI group. The solid curves denote the range of the experimental data and the dashed curves denote an extrapolation based on the Nakagami distribution for the value of m in parenthesis. The standard error of the mean equals σ_M. Additional details are given in Whitney et al.[3]

Our analysis of experimentally obtained amplitude distributions at 137 MHz has shown that there is close agreement with the Nakagami m distribution. It has also been shown[4] that $S_4 = 1/\sqrt{m}$ for $m \geq 0.5$, where S_4 is the notation used by

4. Chytil, B. (1967) The distribution of amplitude scintillation and the conversion of scintillation indices, J. Atmos. Terr. Phys., 29:1175-1177.

Briggs and Parkin[5] and is an exact measure of scintillations. The relation between S_4 and m is given in Figure 5.

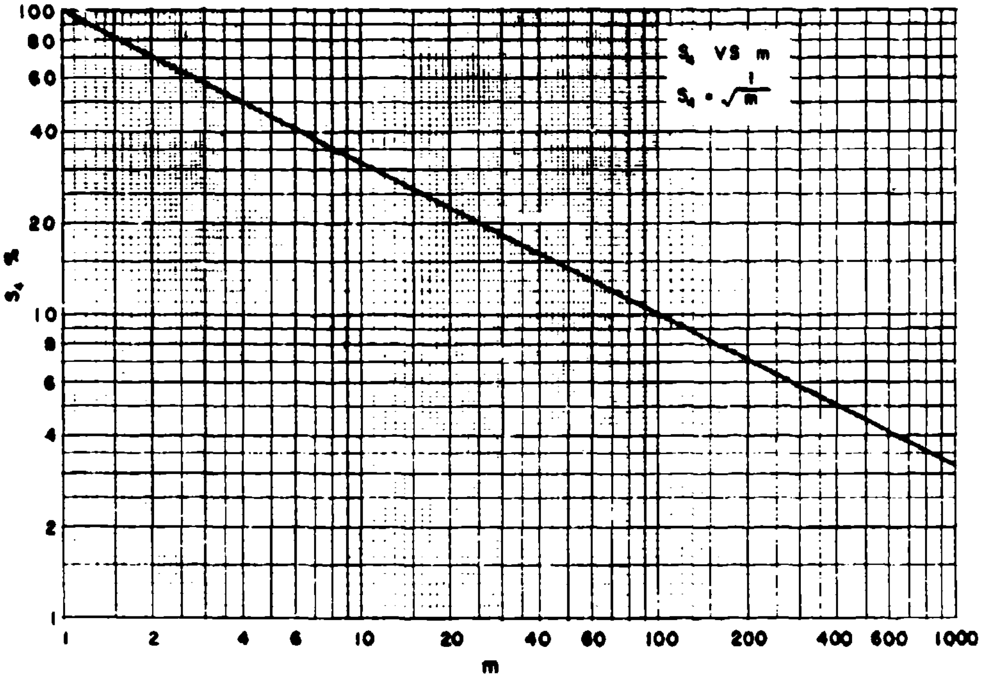

Figure 5. The Relation Between S_4 and m

4. RELATIONSHIP OF SI TO m

Fifty-two 15-min samples of scintillation data were processed by the computer to obtain a cdf and the best fit to a particular m value. The samples were also scaled for SI (dB). A smoothed curve giving the relationship of scintillation index (dB) to the Nakagami m parameter is shown in Figure 6.

Since Figure 1 shows the relationship between SI(dB) and SI(%), Figure 6 can be replotted to give the variation of SI(%) with m as shown in Figure 7.

5. Briggs, B.H. and Parkin, I.A. (1963) On the variation of radio star and satellite scintillations with zenith angle, J. Atmos. Terr. Phys., 25:339-366.

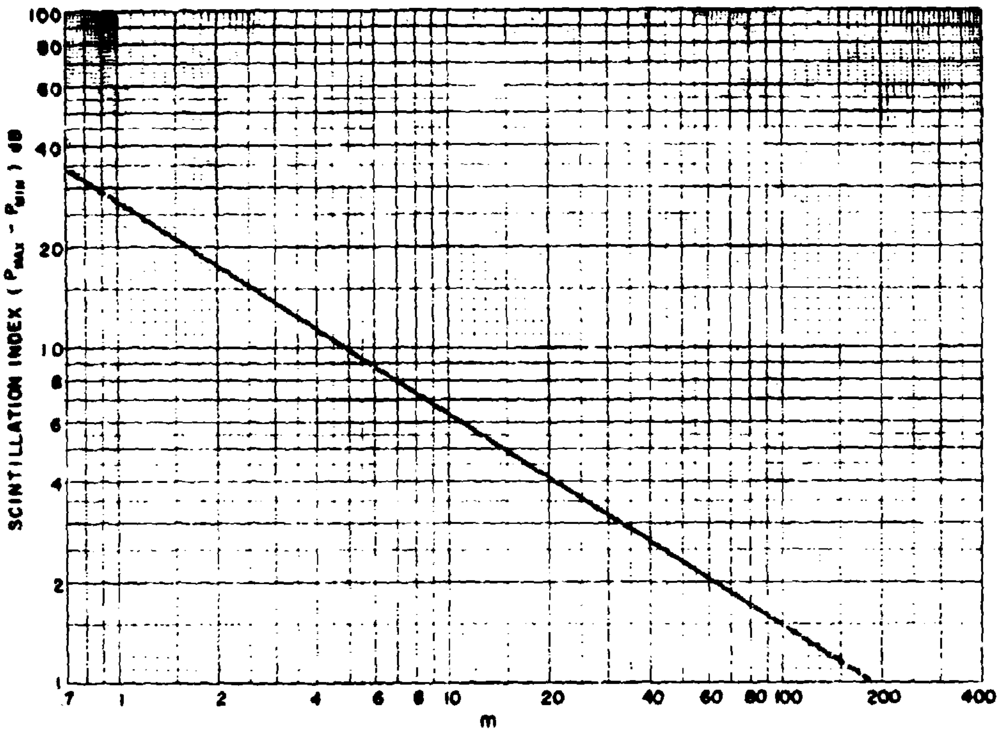

Figure 6. Variation of Nakagami m Parameter with Scintillation Index. m was determined by computer processing of 137-MHz data; SI was obtained by scaling strip charts for fifty-two 15-min samples

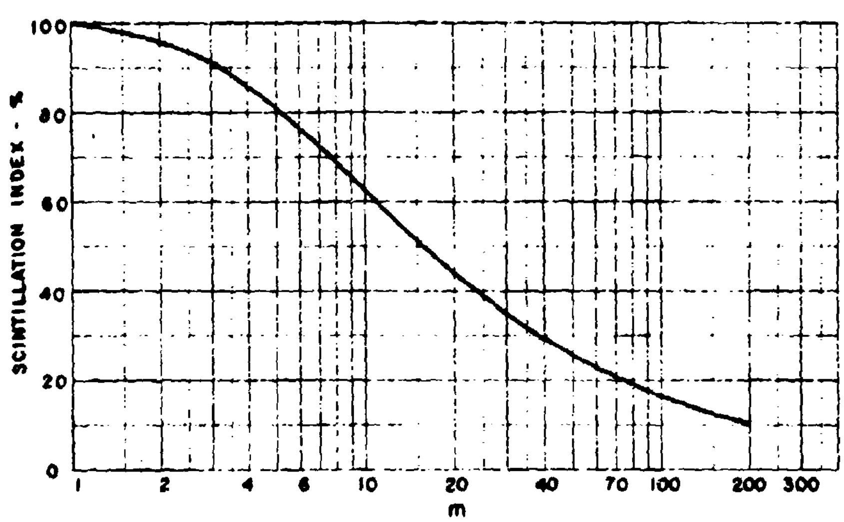

Figure 7. Scintillation Index vs m Taken from Figure 1 and Figure 6

5. RELATIONSHIP OF SI TO PERCENTILE POINTS

Figure 8 shows a histogram of the percentile points of the fifty-two amplitude distributions that had a dB change equal to $P_{max} - P_{min}$. Each of the 52 samples used for Figure 6 had an SI value and an amplitude distribution. The percentile points (plus and minus from the median level) were recorded from the distribution that gave a dB change equal to the scaled value from the chart records. It shows that, in general, the value of SI using the third peak definition agrees with the one percent points on the cdf.

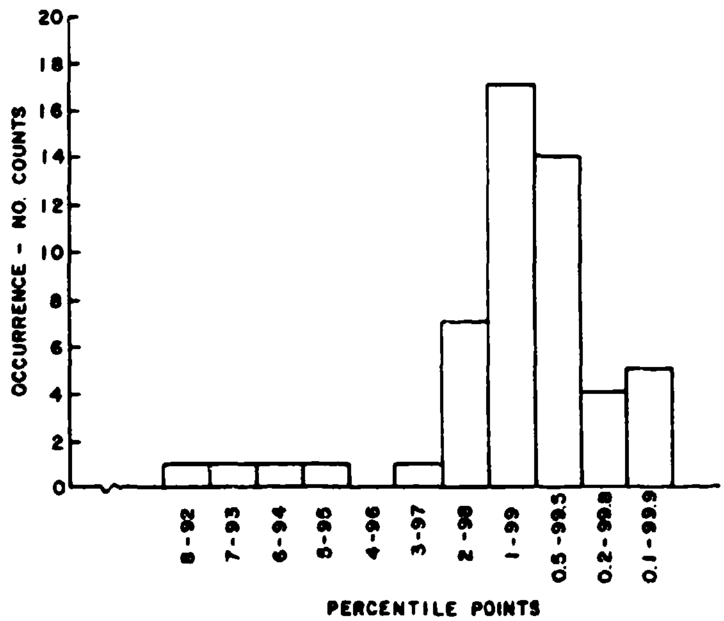

Figure 8. Histogram of the Amplitude Distribution Percentile Points that Correspond to the $P_{max} - P_{min}$ (dB) Levels for the Fifty-Two 15-min Samples in Figure 6

6. RELATIONSHIP OF SI TO FADE LEVELS

The conversion between SI(%) and SI(dB), that is $P_{max} - P_{min}$, was based on equal percentage changes from the average level (Table 1). Since the negative changes or fade levels were recorded for the data for Figure 8, we can check the experimentally determined change against the assumption of equal percentage change.

The results are shown in Figure 9 where the calculated fade level is compared with the measured fade level for various SI(dB) levels. It illustrates that an SI = 8.5 dB (75 percent) would give a 6-dB fade according to the calculated relationship (solid line), but the experimental data would require an SI = 10 dB (82 percent) to give a 6-dB fade.

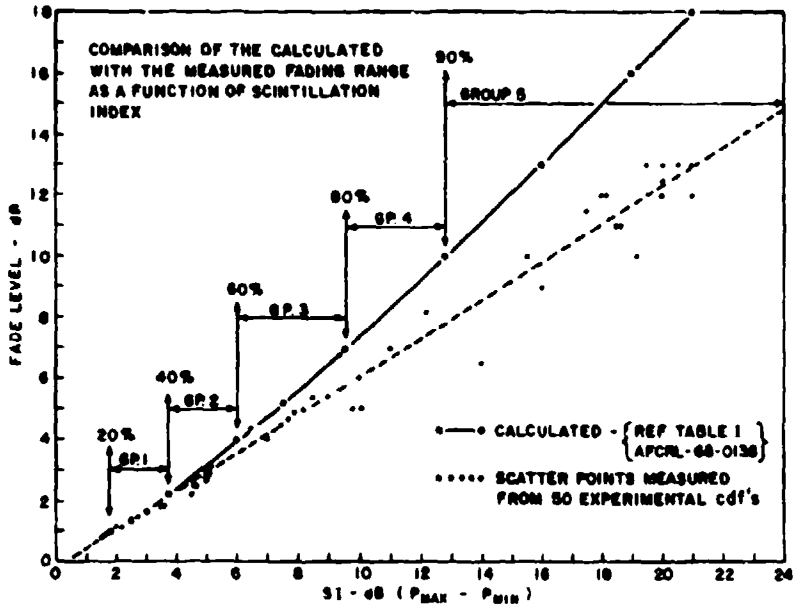

Figure 9. Comparison of the Calculated- with the Measured- Fading Range as a Function of Scintillation Index

7. FREQUENCY DEPENDENCE

It is important to be able to extrapolate the results of scintillation studies at a specific frequency to another frequency of interest. In order to accomplish this, the frequency dependence or spectral index of scintillations must be known. Based on scintillation index S, a spectral index can be expressed as $\eta = [\log (S_1/S_2)]/[\log (f_1/f_2)]$. Since $S = (1/m)^{1/2}$, a spectral index in terms of the m parameter can be obtained as $\eta_m = [\log (m_1/m_2)]/[\log (f_1/f_2)]$. These expressions are valid as long as scintillation index and m are a constant power-law function of frequency.

Satellites ATS-3 and ATS-5 had transmissions at 137 and 412 MHz which allowed the measurements of an m value for 119 simultaneous 15-min amplitude distributions at each frequency. The spectral index was then calculated and the results are shown in the histogram, Figure 10.

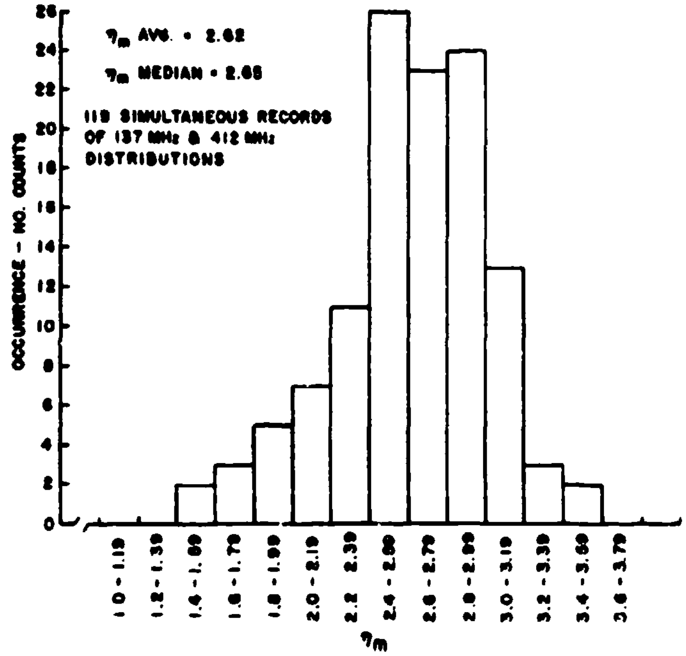

Figure 10. Histogram of Spectral Index η_m

8. USES

8.1 Extrapolation of the Effects of Scintillations to a Desired Frequency

Suppose, for example, a short sample (15 min) of scintillation data is measured at 137 MHz to have a cdf with an m = 1.5 and it desired to estimate the cdf at 254 MHz. Under the assumption that the measured spectral index η_m is a constant power law function of frequency, we can use the average value of 2.62 from Figure 10 to evaluate the following expression:

$$\eta_m = \frac{\log \frac{m_H}{m_L}}{\log \frac{f_H}{f_L}}$$

$$\log \frac{m_H}{m_L} = \eta_m \log \frac{f_H}{f_L}$$

$f_H = 254$ MHz

$f_L = 137$ MHz

$$\log \frac{m_H}{m_L} = 2.62(.268) = .703$$

$m_L = 1.5$

$$m_H \doteq 7.57$$

$\eta_m = 2.62$

The resulting distribution for the calculated m can then be read or interpolated from the theoretical m curves shown in Figure 3.

8.2 Conversion of Occurrence Statistics of SI to a cdf

We have measured 137 MHz scintillations at Narssarssuaq, Greenland, for several years and have selected the following statistics to show the technique for converting to a cdf. The technique has been shown[3] to give sufficient accuracy for engineering purposes by the use of the models which are determined for Hamilton data and shown in Figure 4.

Table 2. Occurrence for Narssarssuaq: February-April 1968-1972

K = 0 - 3	2200 - 0200 LT	
SI Group	SI%	% Occurrence
0	0-19	5.31
1	20-39	8.85
2	40-59	12.48
3	60-79	16.51
4	80-89	11.66
5	≥90	45.18

The percent occurrence of the 15-min scintillation indices are given for the 4-hour local time block and a two-month period for the indicated years under quiet magnetic conditions. To determine a cdf which represents this sorting of SI, the percent occurrence of each SI group is multiplied by its probability at each dB level

as given by the models in Figure 4, and the values are summed to give a composite curve. Expressed mathematically:

$$\mathrm{cdf(dB)} = \sum_{\mathrm{Group}=0}^{5} f(G)\, P(\mathrm{dB}),$$

where f(G) is the frequency of occurrence of a group and P(dB) is the probability or percent of time at each dB level from Figure 4. The dashed curve in Figure 11 then gives the distribution for the occurrence of SI groups listed above.

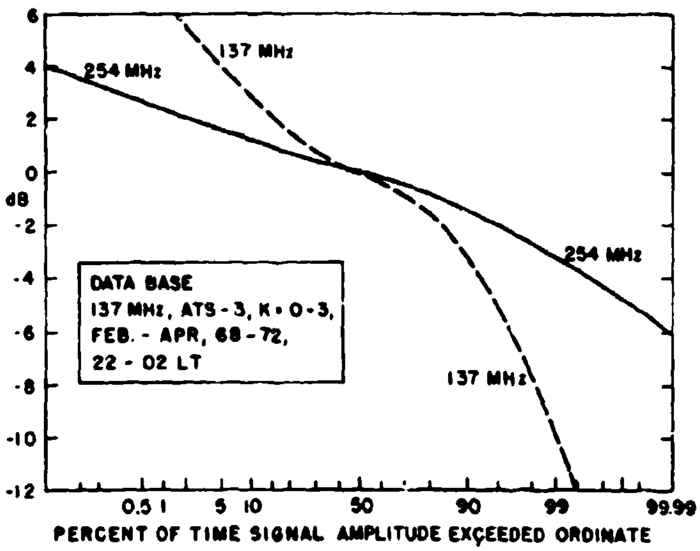

Figure 11. Extrapolated Scintillation Amplitude Distribution Based on Measured Data from Narssarssuaq at 137 MHz

8.3 Given the Occurrence Statistics for 137-MHz SI as Listed in Preceding Section, Find the Expected Distribution at 254 MHz for the Statistics

Again, the assumption is made that the spectral index is a constant power law function of frequency and is given by the average value from Figure 10, that is, $\eta_m = 2.62$. In the same manner that an individual distribution was converted to a

different frequency as in Section 8.1, then all the model distributions of Figure 4 can be converted to their equivalents at 254 MHz by $m_H = 5.05\, m_L \sim 5\, m_L$. The m values for each SI group are listed in Table 3 for 137 MHz and the converted value to 254 MHz.

Table 3. The m Values for Each SI Group

Group	m_L (137 MHz)	m_H (254 MHz)
1	40	200 (negligible)
2	15	75
3	10	50
4	4	20
5	2	10

The percent of time values for each dB level for each m_H curve were read and then multiplied by the percent occurrence of the SI groups to give the resulting distribution at 254 MHz shown by the solid curve in Figure 11.

8.4 Given an S_4 at 400 MHz (Lincoln Laboratory Data), Find the Equivalent AFCRL SI at 254 MHz

Given an S_4 at 400 MHz, we can calculate an m at 400 MHz from $m = 1/S_4^2$ or Figure 5. Assuming that the frequency dependence experimentally determined from 137-MHz and 412-MHz data is valid, the relationship of m_{400} to m_{254} can be found

$$\log \frac{m_{400}}{m_{254}} = \eta_m \log \frac{400}{254} = 2.62(.197) = .517,$$

$$\frac{m_{400}}{m_{254}} = 3.29,$$

$$m_{254} = .304\, m_{400}.$$

The relationship between m at 400 MHz and m at 254 MHz is plotted in Figure 12.

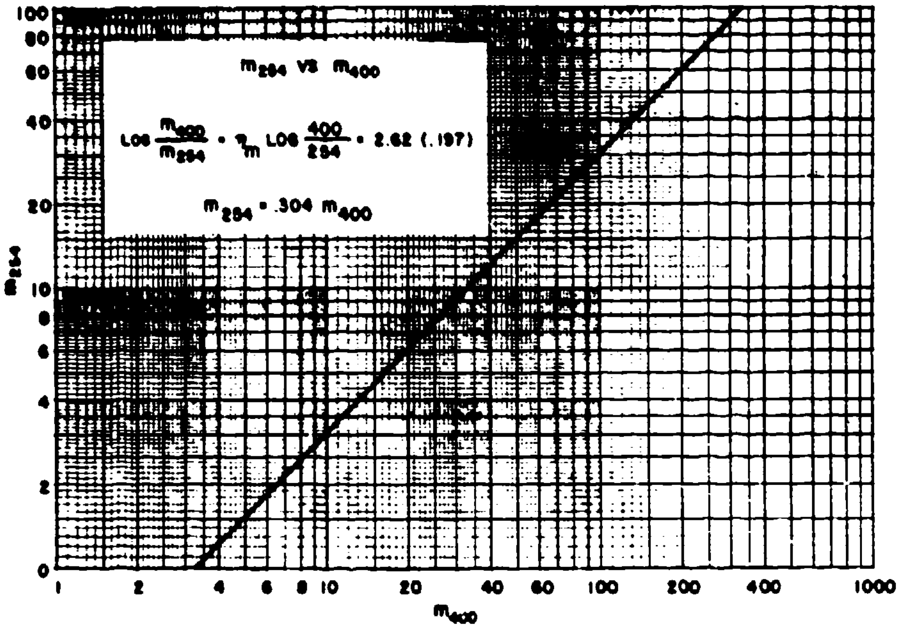

Figure 12. Plot of the Relationship Between m at 400 MHz and m at 254 MHz

We have already related the AFCRL SI to m as shown in Figure 7. This was based on experimental data at 137 MHz. Unless the scatter mechanism changes at other frequencies, it is reasonable to assume that this relationship is valid at 254 MHz. Therefore, from Figures 5, 7, and 12, we can construct the desired relationship, SI_{AFCRL}%(254 MHz) vs S_4%(400 MHz) by the following steps:

(a) given a value of S_4(400), read m from Figure 5;

ex. $S_4 = 10\%$, $m_{400} = 100$.

(b) given m_{400}, read m_{254} from Figure 12;

ex. $m_{400} = 100$, $m_{254} = 30$.

(c) given m, read SI% from Figure 7;

ex. m = 30, SI = 35%.

Therefore, an $S_4 = 10\%$ at 400 MHz corresponds to an SI = 35% at 254 MHz. These steps can then be followed to plot the curve shown in Figure 13.

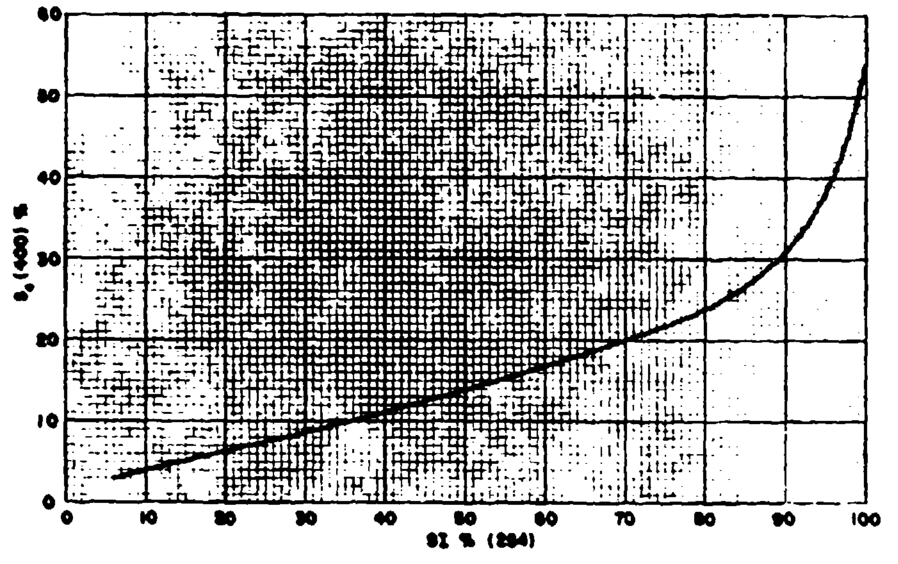

Figure 13. Conversion of 400 MHz SI to 254 MHz

9. CONCLUSIONS

Much of the analysis that has been presented depends on a knowledge of the frequency dependence or spectral index of ionospheric scintillations. A constant power-law function of frequency was assumed for scintillation index and m, and in the 100- to 400-MHz range this may be an error. The evidence of extreme scintillations makes this assumption particularly tenuous for the equatorial regions. Additional data must be acquired at closely spaced frequencies to more adequately define the dependence of scintillations on frequency.

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2. Whitney, H.E., Aarons, J. and Malik, C. (1969) A proposed index for measuring ionospheric scintillations, Planet. Space Sci., **17**:1069-1073.
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4. Chytil, B. (1967) The distribution of amplitude scintillation and the conversion of scintillation indices, J. Atmos. Terr. Phys., **29**:1175-1177.
5. Briggs, B.H. and Parkin, I.A. (1963) On the variation of radio star and satellite scintillations with zenith angle, J. Atmos. Terr. Phys., **25**:339-366.

Appendix A

Instruction for Completing FORTRAN Coding Form for Scaling 15-min Scintillations (ATS-3)

Scintillations are scaled from the chart as a decibel change corresponding to $P_{max} - P_{min}$ where P_{max} is the third peak down from the maximum and P_{min} is the third null up from the lowest excursion. The values for each 15-min period are recorded in 3 columns (tens, units, and tenths of dB) starting with column 12. The scintillations are measured from the calibration steps as a relative change in decibels. Each card is a 4-hour block, indexed by column 7. Use -99 to record interference or inability to make a measurement in any 15-min period. Use 001 to record signal present but scintillations too small to read, that is, scintillations less than 1 dB. When there is interference or poor data during an entire 4-hour block, put a 9 in column 10. When signal is present but scintillations are too small to read during an entire 4-hour block, put a 1 in column 10; the remainder of the line will be left blank. If data is missing for a full 4-hour period, no card is required. If data is missing or poor for a portion of a 4-hour period, -99 is required in the pertinent columns.

15-MIN SCINTILLATION FORMAT

Col. 1, 2	Year	
3, 4	Month	
5, 6	Day	
7	4-hour period	1 = 00 - 04 2 = 04 - 08 3 = 08 - 12 4 = 12 - 16 5 = 16 - 20 6 = 20 - 24
8, 9	Blank	
10	9 = interference for 4 hours 1 = scintillations less than 1 dB for 4 hours	
11	Blank	
12, 13, 14	1st 15-min scintillation reading in dB 12 = tens dB, 13 = units dB, 14 = tenths dB	
15, 16, 17	2nd 15-min reading	
etc., to	etc., to	
57, 58, 59	12th 15-min reading	
60-67	Blank	
68, 69	Source: A1 = ATS-1, A3 = ATS-3, A5 = ATS-5, L6 = LES-6 I2 = Intelsat 2 F-2	
70, 71, 72	Frequency	
73, 74	Station No.	
79, 80	dB	